Characterising Secondary School Mathematics Lessons Using Teachers' Pedagogical Concept Maps

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The construction of concept maps is increasingly seen as a useful tool for mathematics educators. The maps have usually been built either directly by the individual themselves or from their comments in a discussion or an interview. In this paper we describe how a novel kind of concept map was constructed based on questions teachers use in their classroom lessons. An analysis of what we call *pedagogical concept maps* may assist teachers to construct and determine the goals of a lesson. We present here the results of analysing four lessons from a conceptual and procedural perspective which show their value in discriminating between these kinds of lessons.

Introduction

One of the major roles of teachers is to act as facilitators of student knowledge, assisting them to build the relational or conceptual understanding which will enhance learning prospects (Skemp, 1976, 1986; Hiebert & Lefevre, 1986). However, it is evident that the quality of this teacher guidance will be determined by the teacher's own content knowledge, their pedagogical content knowledge (Shulman, 1986), and by how they structure their classroom teaching environment (see e.g., Bromme, 1994; Chinnappan & Thomas, 1999; Ball, 2000). This study used the hypothesis that the planning of classroom activities and the informal questions asked by teachers in the delivery of a lesson are likely to be influenced by their content and pedagogical knowledge structures. This assumption is based on the idea that teachers follow a hypothetical learning trajectory in their teaching (Simon, 1995) based on the content knowledge they have built themselves. Moreover, it also seems reasonable to assume that the relationships exhibited by the teachers' knowledge structures are very close to the ones that they want to encourage students to build in their own minds.

We believe that in creating a classroom environment teachers' informal questioning plays a vital role. We have previously reported on a framework that we have constructed to help identify and classify the types of informal questions that teachers employ (Liyanage, Irwin, & Thomas, 2000). Our intention here is to identify the informal assessment questions (and other teacher utterances) used in mathematics lessons with the teacher's schematic content knowledge in that particular domain, and hence with the pedagogical content knowledge garnered to deliver the lesson. Thus our aim was to represent this knowledge by means of novel concept maps (Novak & Gowin, 1985, Shavelson, 1998), which we have designated teachers' pedagogical concept maps, and to use these to provide information on the goals of the lesson, especially from the perspective of the procedural or conceptual basis of the nature of the teaching strategy.

Concept Maps: Assessing Internal Organisation of Knowledge

Concept maps have been considered by a number of researchers (e.g., Novak & Gowin, 1985; Shavelson, Lang, & Lewin, 1993; Ruiz-Primo, Schultz, Li, & Shavelson, 1998;

Nicoll, Francisco, & Nakhleh, 2001) as an important systematic device for depicting internal organisation or structure of an individual's knowledge. These representations display relationships between concepts, ideas, theories, procedures, and other mathematical entities. They are usually represented as comprising nodes with connecting lines, which may be labelled, with the labels describing the basis on which the connections have been made. They have been used in diverse ways, such as to assess students' conceptual knowledge (Williams, 1998), and to characterise teachers' mathematical content knowledge (Chinnappan, Lawson, & Nason, 1999), and in a number of subject areas, including physics (Austin & Shore, 1995), political science (Parkes, Suen, Zimmaro, & Zappe, 1999), statistics (Roberts, 1999), and mathematics (Bolte, 1999). In each case they have proven to be an effective way of looking at the organisation and the structure of a person's knowledge in a particular subject area, and are considered to be able to reflect conceptual understanding, or its lack (Williams, 1998).

Following a review of the literature, Shavelson, Lang, and Lewin (1993) have documented wide variations in concept mapping techniques, with no less than 128 identified variations. One important variation is that concept maps can be drawn by the person him/herself using techniques taught by the researcher in a training session (e.g., Williams, 1998) or by a second person using the information collected in clinical interviews (e.g., Chinnappan, Lawson & Nason, 1999).

Our motivation for the construction of the concept maps was to enable teaches to ascertain the extent to which they have been presenting mathematics procedurally or conceptually. It is important that as teachers we get a feel for the balance between these two vital parts of learning mathematics, and while we could give many examples of why this is so, the research on derivatives by Ferrini-Mundy and Graham (1994) clearly supports it. They concluded that "Startling inconsistencies exist between performance, particularly on procedural items, and conceptual understanding." (p. 43), giving a graphic example of a student who described in detail an algorithm for sketching the graph of a function including how the first derivative may be used. However, less than one minute later when asked how derivatives are linked to tangents to curves she replied "I'm not exactly sure... I can't remember exactly how it's related to the derivative...I remember doing it, but I can't remember exactly how." (p. 44).

Method

The method used to construct the teachers' *pedagogical concept maps* in this study differs in a major way from the maps used in previous studies. We have developed a method based on our hypothesis that the planning of teaching activities and the informal questions asked during a lesson are highly likely to be influenced by teachers' content and pedagogical content knowledge. Hence we identified teachers' informal assessment questions (and other statements) from the transcripts of videotaped lessons, and employed these to construct concept maps of the mathematical ideas being developed in the lesson. In our construction the labelling of connections between nodes was not carried out because it was not required in our analysis. We were primarily interested in assessing the richness of the resulting schematic structure rather than the manner in which it had been constructed. Hence only quantitative measures were applied to the links and to the resulting structure.

We took four lessons from four different teachers to construct the maps. The teachers, one male (V) and two females (B and E) from New Zealand, and a male (M) from Sri Lanka, are part of a larger study on the use of informal questions in the classroom. The

three New Zealand teachers all taught in co-educational state schools, while the Sri Lankan taught in a private boys' school. The lessons were on logarithms, the laws of logarithms, the normal distribution and rounding numbers to significant figures.

Results

The following algorithm was followed in drawing the pedagogical concept maps.

Step 1: The main theme of the lesson was identified by looking at the title that the teacher put up on the board or his/her declarations, and this was then drawn in the centre of the map.

Step 2: The concepts, procedures, processes, objects, definitions, statements and the other ideas that the teacher tried to relate *directly* to the main theme during the lesson were identified from the informal questions asked and other classroom activities. For example, by asking the question, "what do we do when we have to multiply numbers with indices" during a lesson on "the laws of logarithms" the teacher is attempting to a make a connection between laws of logarithms and laws of indices in the students' minds. Likewise there can be many other ideas that the teacher tries to connect *directly* to the main theme of the lesson. These were drawn as nodes surrounding the main theme of the lesson and the links identified were drawn using arrows.

Step 3: Other concepts, definitions etc. that were mentioned in the lesson but were related to the nodes surrounding the main theme rather than the main theme itself, were identified. For example, standard form for numbers, mathematical operations, and square root of numbers are some of the nodes secondary to the main theme of logarithms. These nodes were linked to other appropriate nodes.

Step 4: Practical applications and mathematical solutions to given problems were identified and linked to other appropriate nodes.
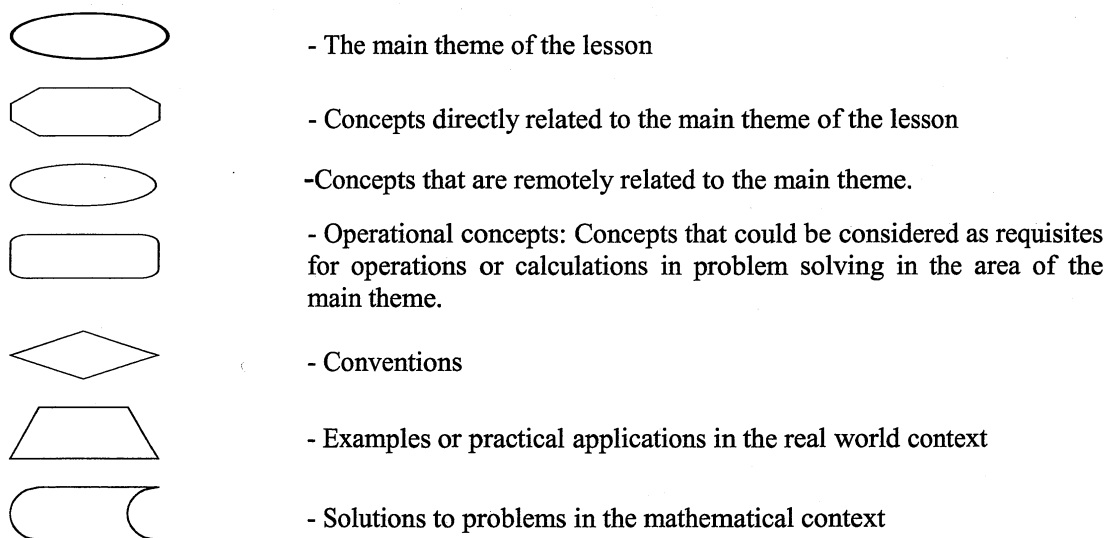

- The main theme of the lesson

- Concepts directly related to the main theme of the lesson

-Concepts that are remotely related to the main theme.

- Operational concepts: Concepts that could be considered as requisites for operations or calculations in problem solving in the area of the main theme.

- Conventions

- Examples or practical applications in the real world context

- Solutions to problems in the mathematical context

Figure 1. The meaning of symbols used in drawing the concept maps.

In concert with the above steps, different shapes were used to denote the main theme, concepts directly related to the main theme, conventions, solutions in the real world context, and the solutions in the mathematical context etc. (see Figure 1). However, these different symbols are purely for ease of reading of the maps and do not play a direct role in the quantitative analysis. Using the process described above, pedagogical concept maps for lessons from four teachers were drawn, two of which are shown in Figures 2 and 3 (where the curved lines represent cross-links). The following examples of teacher comments and questions from the lessons are presented to illustrate how the maps were constructed. The extraction of the data proved a relatively difficult process, and one in which we tried to

remain objective, taking only explicit linkings in context rather than attempting to read implied connections into the teacher comments and questions. Ideally we would want to interview the teachers and obtain their comments on the lesson and the concept maps, but on this occasion this was not possible.

Mr M's lesson (S is used to represent a student comment)

M: When we have to write 8 or 100 as logarithms, what do we need?

S: A base   Relates base to logarithms

M: Yes, a base. What is the base we normally use?

S: 25.83

M: Tell it to me in standard form   Relates standard form to logarithms

S: $10^1 x 2.583$

M: What is the log of it?

M: What should we do when we divide numbers with indices?

S: We subtract the indices   Use of laws of indices in division of numbers

M: Yes. According to the laws of indices we should subtract them.

Mr V's lesson

V: What about 0.00085. Is it a decimal?   Concept of a decimal number

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V: 35.5283. There are how many decimal places in this number?

S: Four   Concept of decimal places linked to rounding

V: Four. How would you round this number to one decimal place? What is the answer?

Ms E's lesson

E: Some of these other ones [graphs on the board] are quite spread out. What is our measure of spread?

S: Standard deviation   Relates standard deviation to spread

E: Standard deviation. What would you tell me about the SD of this one [pointing to a graph]?

E: You know the total area is one. What is the area here? [points to the shaded RHS of a normal distribution curve]   Relates area to probability and SD

S: 0.5

E: Yes. The table shows all the probabilities of the right hand side. They go up to 0.5. If they go one standard deviation either side the total area would be how much? [shades the area]

Quantitative Analysis of the Concept Maps

While even a cursory examination of Figures 2 and 3 will reveal the differential richness of the two maps in terms of numbers of elements and connections, in our analysis we wanted to construct a more objective quantitative score for the concept maps for future analysis, and in order to be able to form a validation comparison with the results obtained from the Framework for Informal Assessment Questions (FIAQ) previously developed (see Liyanage, Irwin, & Thomas, 2000). The scoring procedure used was based on that formulated by Chinnappan, Lawson, and Nason (1998, 1999) to characterise teachers'

mathematical content knowledge, since it has been shown to be useful in representing the connectedness of teacher knowledge. Following them, two main themes were considered.

1. The 'Richness' of the concept maps: Two basic aspects were considered in defining 'richness', namely, the number of nodes, and the number of links emanating from each node. The higher the number of nodes and links, the more the connections in the concept map and hence richness as a function of connectivity. By also taking the relative importance of each concepts and its relationships into account, a score was awarded to each node type and each type of link (see Table 1).

2. 'Connectedness' in the concept map: Apart from direct links between nodes, three further aspects were considered in defining the connectedness of the concept maps.

   • Cross-links: The cross-links are the 'horizontal' links between nodes that are already connected 'vertically' (by extended connections from the main theme) to other nodes. The importance of cross-links lies in the potential they have for connecting mathematical concepts and ideas, which otherwise could remain separate. Hence they make the schemata richer in inter-conceptual connections.

   • Depth: Some (e.g., Liu, 1994) refer to this as the hierarchies of nodes. However, we take depth as the vertical extension of a node from the main theme on a single path. This shows how far the teacher moves from the main theme in order to make connections between relevant (and closely related) mathematical concepts and the main theme of the lesson.

   • Cycles: A cycle in a concept map is a closed trail along links passing through nodes. They are mediated by a combination of cross-links and depth of links and the directed nature of the links. We refer to the cyclic connectedness of nodes, where cycles can be 3-cycle (3C), 4-cycle (4C), ...etc, according to the number of nodes they pass through, and they are scored thus.
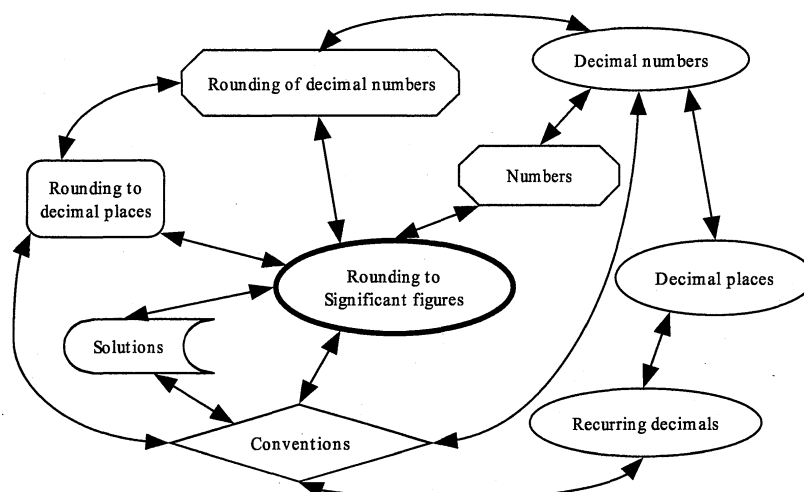

Figure 2. The pedagogical concept map of Mr. V for the lesson on rounding.

Taking into account the above criteria, a scoring procedure generating a numerical value to represent the richness of the concept maps was established. The number of nodes and links was multiplied by a base score to give a weighted total. The base scores applying to nodes, links, cross-links and depth are described in Table 1. The main theme was given the highest base score of 5 since having a focus is a fundamental requirement of any

lesson. Similarly the concepts directly related to the main theme and the practical applications were considered to be next in importance and hence rated a weighting of 3.
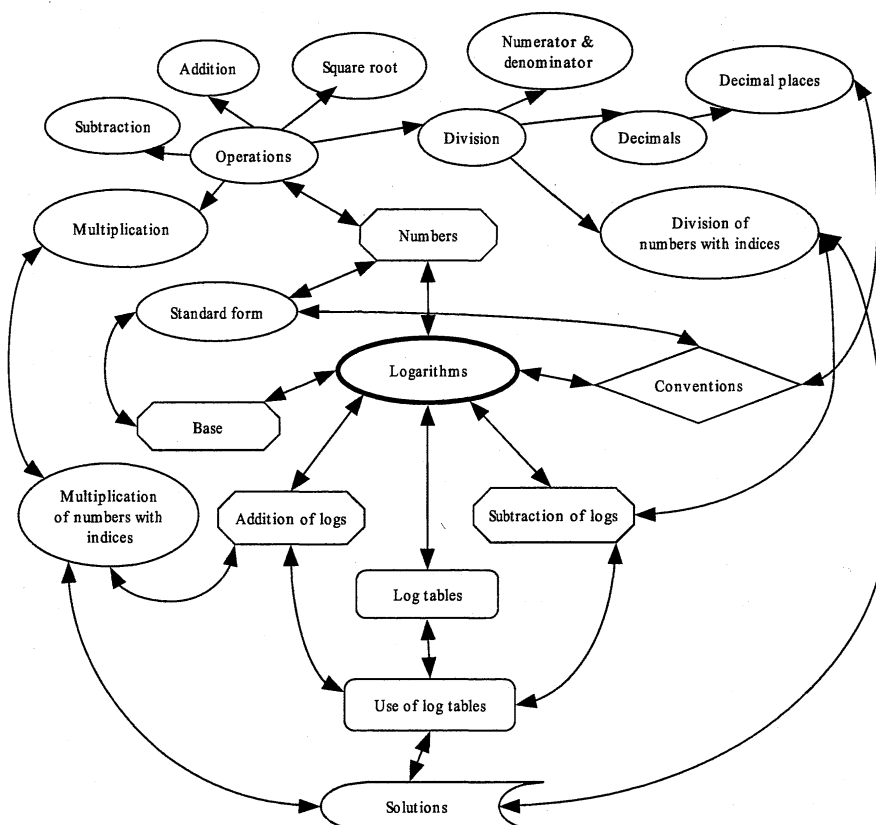

Figure 3. The pedagogical concept map of Mr. M for the lesson on logarithms.

Mathematical procedures (weighting 2) placed next, followed by concepts only remotely related to the main theme (base score weighting 1). The base scores for links were obtained by using the same order of priority (see Table 1). As stated above, the 'connectedness' of the concept maps was considered in two distinct ways. The 'depth' of the map was scored according to its level, one point for each, cycles (mediated by cross-links) were granted a score according to the number of nodes they pass through.

We can see from Table 1 that the quantitative measures obtained from the lessons of Mr V and Mr M were 139 and 236 respectively, and the corresponding values for Ms. B's and Ms. E's lessons were 58 and 178 respectively. On this evidence it seems to be possible to make a distinction between the lessons of Mr M and Ms E and those of Mr V and Ms B. Although it may not be easy to establish a clear-cut dividing line, Mr M and Ms E have higher scores and those above a guideline of around 150 may indicate a more conceptual lesson, and lower scores a more procedural one. The conclusions reached by examining the scores generated by this process were consistent with those from the FIAQ, suggesting that this is a viable method of discriminating between lesson types. It should be remembered that the FIAQ was constructed on the basis of procedural-conceptual distinctions in questioning, as well as inter-representational linking (Liyanage, Irwin, & Thomas, 2000). The FIAQ employs Levels 1 and 2 to measure knowledge questions and those involving simple procedures, while Levels 3, 4 and 5 consider connections between concepts and representations. Mr V had 12 Level 1 and 35 Level 2 questions in his lesson, but only one question at Level 3, while Ms B had 2 Level 1 and 28 Level 2, with 2 Level 3 and only 3 Level 4 questions. In contrast Mr M's totals were 20 Level 1, 7 Level 2, 24 Level 3 and 4

Level 4 and Ms E's were 21 Level 1, 26 Level 2, 11 Level 3 and 7 Level 4. These latter two lessons are clearly differentiated as having a stronger conceptual base by their Level 3 and 4 questions on the framework, as confirmed by the concept maps.

Table 1

The Scoring Procedure and Scores for Teachers' Pedagogical Concept Maps

Features of the lesson	Richness			Connectedness										
	Nodes	Links	Cross links	Depth						Cycles				
				L2	L3	L4	L5	L6	L7	3C	4C	5C	6C	≥7C
1. The theme of the lesson	*5* (5,5)	*3* (15,18)	-											
2. Concepts/procedures directly related to the main theme	*3* (6,12)	*2* (6,12)	*2* (4,10)											
3. Operational concepts or processes	*2* (2,4)	*1* (1,4)	*2* (4,4)	*1*	*2*	*3*	*4*	*6*	*6*	*1*	*2*	*3*	*4*	*5*
4. Other related concepts or procedures	*1* (3,12)	*1* (5,11)	*1* (3,9)	0 / 1	0 / 10	3 / 6	0 / 4	0 / 0	0 / 0	3 / 0	10 / 8	21 / 0	16 / 24	20 / 70
5. Conventions considered in the lesson	*2* (2,2)	*1* (2,1)	*2* (3,4)											
6. Practical applications with solutions	*3* (0,0)	*2* (0,0)	*1* (0,0)											
7. Solutions in the mathematical context	*2* (2,2)	*1* (2,1)	*1* (1,2)											
Totals	(20,37)	(31,47)	(15,29)	03,21						70,102				

Numbers in bold italics give the base score weightings. Other numbers in parentheses and below italic weightings give the scoring for the maps of Mr V and Mr M respectively.

Of course there are still steps necessary to refine the process of constructing *pedagogical concept maps* and using them to gather information about the pedagogy of classroom lessons. One issue is checking the validity of the maps drawn, for example by using a reliability assessor to construct them independently. Also the weightings applied to the various measures of richness could be argued, affecting the differentiation of lessons in the middle ground. In addition it needs to be said that the approach may not prove suitable for all types of lessons. Those we have observed were primarily instructive in nature and it has to be recognised that many teachers value a constructivist approach which may lead them away from giving direct guidance even in the form of the type of questioning seen here, preferring to allow students to interact in investigative situations. It will be interesting to see if the method can be usefully applied to the teacher's role in these types of lessons. Another consideration is that the content of some lessons might better lend itself to the introduction of concepts than others, thus skewing the richness of the maps. This is an area that needs some investigation. Having said this we believe that many teachers value both the importance of teaching concepts as well as procedures and the opportunity to engage in constructive self-analysis of their teaching practice. It may be that the concept mapping techniques described here will give them another instrument to be able to do so.

References

Austin, L. B., & Shore, B. M. (1995). Using concept maps for assessment in physics. *Physics Education, 30*(1), 41-45.

Ball, D. L. (2000). Bridging practices: Intertwining content and pedagogy in mathematics teaching and learning. In J. Bana & A. Chapman (Eds.), *Mathematics Education Beyond 2000* (Proceedings of the Twenty-third annual conference of the Mathematics Education Research Group of Australia, pp. 3-10). Fremantle, Australia: MERGA.

Bolte, L. A. (1999). Using concept maps and interpretive essays for assessment in mathematics. *School Science & Mathematics, 99,* 19-30.

Bromme, R. (1994). Beyond subject matter: a psychological topology of teachers' professional knowledge. In R. Biehler, R. W. Scholz, R. Strässer, & B. Winklemann, (Eds.), *Didactics of Mathematics as a Scientific Discipline* (pp. 73-88). Dordrecht: Kluwer Academic Publishers.

Chinnappan, M., Lawson, M., & Nason, R. (1999). The use of concept mapping to characterise teachers' mathematical knowledge. In J. Truran & K. Truran (Eds.), *Making the Difference* (Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australia, pp. 167-176). Adelaide, Australia: MERGA.

Chinnappan, M., & Thomas, M. O. J. (1999). Structured knowledge and conceptual modelling of functions by an experienced teacher. In J. Truran & K. Truran (Eds.), *Making the Difference* (Proceedings of the 22nd annual conference of the Mathematics Education Research Group of Australia, pp. 159-166). Adelaide, Australia: MERGA.

Ferrini-Mundy, J., & Graham, K., (1994). Research in calculus learning: Understanding of limits, derivatives, and integrals, In J. J. Kaput, & E. Dubinsky, (Eds.), *Research issues in undergraduate mathematics learning* (pp. 31–45), Mathematical Association of America.

Hiebert, J. & Lefevre, P. (1986). *Conceptual and procedural knowledge: The case of mathematics.* London: Lawrence Erlbaum Associates.

Liyanage, S., Irwin, K., & Thomas, M. O. J. (2000). Informal assessment used by secondary school mathematics teachers. In J. Bana & A. Chapman (Eds.), *Mathematics Education Beyond 2000* (Proceedings of the Twenty-third annual conference of the Mathematics Education Research Group of Australia, pp. 377-384). Fremantle, Australia: MERGA.

Liu, X. (1994). *The validity and reliability of concept mapping as an alternative science assessment when item response theory is used for scoring.* Paper Presented at the Annual Meeting of the American Educational Research Association. New Orleans, LA.

Nicoll, G., Francisco, J., & Nakhleh, M. (2001). A three-tier system for assessing concept map links: A methodological study. *International Journal of Science Education, 23*(8), 863-875.

Novak, J. D. & Gowin, D. B. (1985). *Learning how to learn.* New York: Cambridge University Press.

Parkes, J. T., Suen, H. K., Zimmaro, D. M., & Zappe, S. M. (1999). *Structural knowledge as a pre-requisite to valid performance assessment scores.* Paper Presented at the Annual Meeting of the National Council on Measurement in Education. Montreal, Canada.

Roberts, L. (1999). Using concept maps to measure statistical understanding. *International Journal of Mathematical Education in Science & Technology, 39,* 707-717.

Shavelson, R. J., Lang, H., & Lewin, B. (1993). *On concept maps as potential "authentic" assessment in science. indirect approach to knowledge representation of high school science.* ERIC Document No. ED 367691.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15,* 4-14.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26,*(2), 114-145.

Skemp, R. R. (1976). Relational understanding and instrumental understanding, *Mathematics Teaching, 77,* 20-26.

Skemp, R. R. (1986). *The psychology of learning mathematics.* Harmondsworth: Penguin.

Ruiz-Primo, M. A., Schultz, S. E., Li, M., & Shavelson, R. J. (1998). *Comparison for the reliability and validity of scores from two concept-mapping techniques.* ERIC Document: ED: 422378.

Williams, C. G. (1998). Using concept maps to assess conceptual knowledge of function. *Journal for Research in Mathematics Education, 29,*(4), 414-421.